

Security requirement: Without K, decryption must be computationally infeasible

4.2 Hardness & Key Space

- **Brute force:** Try all possible keys
- **Bits of security:** $\log_2(|\text{keyspace}|)$
- Example: Caesar cipher \rightarrow 25 keys \rightarrow 4.6 bits \rightarrow insecure
- Substitution cipher: $26! \approx 4 \times 10^{28}$ keys (\approx 88 bits) \rightarrow still breakable via frequency analysis (statistical attack)
- **Cryptanalysis:** Exploiting structure or frequency of plaintext to reduce key search space

4.3 Key Terms

- **Encryption algorithm:** Deterministic or randomized transformation parameterized by K
- **Decryption algorithm:** Inverse function using K
- **Key space:** All possible keys, defines theoretical security upper bound
- **Security level:** Effort (2^n operations) required for best known attack

5 Adversary Types

5.1 Passive Eavesdropper

Eve only reads ciphertext \rightarrow limited model (e.g., substitution cipher).

5.2 Known Plaintext Attack (KPA)

- Eve knows pairs $(M, C = E_K(M)) \rightarrow$ infers key patterns
- Realistic: headers or predictable data leak info (e.g., "From:" field)

5.3 Chosen Plaintext Attack (CPA)

- Eve can choose messages and get encrypted outputs (encryption oracle)
- Stronger than KPA \rightarrow full break for substitution cipher (choose "abcdefghijklmnopqrstuvwxyz")

5.4 Side-Channel Attacks

- Exploit physical information during encryption: timing, power use, electromagnetic leaks
- Common on devices holding third-party keys (e.g., smart cards, DRM)

6 One-Time Pad (OTP)

6.1 Principle

- Key = truly random bits, same length as message
- Encryption: $C = M \oplus K$; Decryption: $M = C \oplus K$
- Guarantees perfect secrecy: C gives zero information about M

6.2 Conditions for Perfect Secrecy

Key must be: random, as long as message, used only once.

Reusing key \rightarrow breaks secrecy ($C_1 \oplus C_2 = M_1 \oplus M_2$).

6.3 Integrity Flaw

- OTP ensures confidentiality only
- Bit-flip in ciphertext flips same bit in plaintext \rightarrow no integrity protection

6.4 Practical Limitation

- Key distribution problem: hard to securely share large random keys
- OTP ideal but impractical for real-world use

7 Stream Ciphers

7.1 KSG Principle

A stream cipher mimics OTP: it generates a pseudo-random stream $S = \text{KSG}(K, IV)$ from a secret key K and a public IV.

- Both sides recompute the same S to encrypt/decrypt with XOR
- The IV changes for each message so S differs every time, avoiding key reuse
- Knowing IV or the KSG algorithm does not reveal K if the cipher is secure
- Reusing IV or breaking K compromises all messages

7.2 Idea

- Emulate OTP using short key + keystream generator (KSG)
- Inputs: secret key K + public IV \rightarrow pseudo-random stream S
- Encryption: $C = M \oplus S$; Decryption: same operation

7.3 Properties

- Symmetric key: same K for encryption/decryption
- IV: ensures two messages use different keystreams
- Security: S must be computationally indistinguishable from random

7.4 Pros / Cons

- **Pro:** Fast, low memory, low error propagation
- **Cons:** Low diffusion \rightarrow bit-level tampering easy (no integrity)
- **Cons:** Vulnerable if keystream repeats (periodicity)

7.5 Attacks & Flaws

- Finite KSG state \rightarrow eventually periodic stream
- Short period \Rightarrow pattern repetition \Rightarrow message recovery
- Linear designs (e.g., LFSR) predictable \rightarrow broken (A5/1 GSM)

8 Public Key Cryptography

8.1 Diffie-Hellman Key Exchange

- Solve key distribution: allow two parties to agree on a shared secret over an insecure channel
- Public parameters: prime p , generator g
- Alice \rightarrow picks a , sends $A = g^a \mod p$
- Bob \rightarrow picks b , sends $B = g^b \mod p$
- Shared secret: $K = g^{ab} \mod p = (B^a = A^b \mod p)$
- Eve knows (A, B, g, p) but cannot recover $a, b \rightarrow$ Discrete Log Problem

8.2 Security & Limits

- Security = hardness of computing discrete log

- Provides key agreement, not authentication \rightarrow vulnerable to man-in-the-middle
- Solution: add digital signatures or certificates (CA) to verify identities
- Examples of related systems: RSA (factoring), ECC (elliptic-curve discrete log), post-quantum (lattice)

9 Authenticity

9.1 Public Key Cryptography

- Each user owns public key (PK) and secret key (SK)
- Anyone can encrypt with PK \rightarrow only SK decrypts
- Enables confidentiality without shared secret
- Needs trusted Public Key Infrastructure (PKI) to bind identities to keys

9.2 Digital Signatures

- Sign: $S = \text{Sign}_{SK}(M)$
- Verify: $\text{Verify}_{PK}(M, S) \rightarrow$ true if valid
- Ensures authenticity, integrity, and non-repudiation
- Forgery infeasible without SK

9.3 Hash-Based Signing

- Instead of signing M directly, sign $H(M)$ (faster, smaller)
- Required hash properties:
 - Second pre-image resistance: can't find $M' \neq M$ with same hash
 - Collision resistance: can't find any (M, M') with same H
- Pre-image resistance less critical (M often public)

9.4 Hash Functions

- Input any length \rightarrow fixed short digest
- Core properties:
 - Pre-image: given H, can't recover M
 - Second pre-image: given M, can't find M' with same H
 - Collision: can't find any pair (M, M') with same H
- Use: SHA-2, SHA-3. Avoid MD5, SHA-1
- Applications: signatures, HMACs, password storage, integrity checks

10 Block Ciphers

10.1 Principle

- Process data in fixed-size blocks (e.g., 128 bits)
- Use same secret key K for encryption and decryption
- Deterministic mapping: $C = E_K(M)$, $M = D_K(C)$
- Example: AES (Advanced Encryption Standard)

10.2 Goal & Limitation

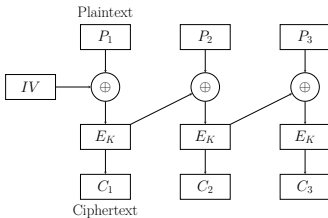
- Acts like a permutation over all possible blocks
- Deterministic \rightarrow same plaintext block \Rightarrow same ciphertext \Rightarrow pattern leaks
- Solution: use modes of operation with randomization (IV)

10.3 Block Cipher Modes

10.3.1 ECB – Electronic Code Book

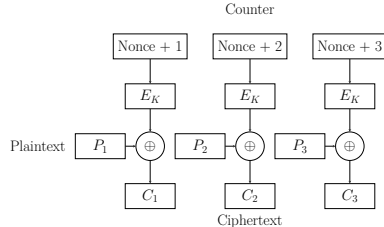
- Encrypt blocks independently: $C_i = E_K(M_i)$
- Leaks identical patterns (e.g., image structure)

10.3.2 CBC – Cipher Block Chaining



- Adds diffusion with XOR chaining
- Hides patterns, needs random IV
- Sequential encryption (not parallelizable)

10.3.3 CTR – Counter Mode



- Turns block cipher into stream cipher
- Parallelizable, random access, requires unique nonce per key
- Same nonce reuse \Rightarrow catastrophic ($C_1 \oplus C_2 = M_1 \oplus M_2$), like OTP reuse

10.3.4 Summary

- ECB: insecure, reveals structure
- CBC: secure if IV random; sequential
- CTR: fast, parallel, secure if nonce unique

11 Authenticated Encryption & Integrity

11.1 Why Authentication Matters

- Confidentiality alone \neq security
- Need to detect if ciphertext was modified (tampering, replay)
- Combine encryption + integrity protection

11.2 MAC – Message Authentication Code

- Symmetric integrity check: Tag = $\text{MAC}_K(M)$
- Verification: recompute $\text{MAC}_K(M)$ and compare
- Detects modification and confirms origin (shared key)
- No third-party proof (no non-repudiation)
- Examples: HMAC-SHA256, CMAC-AES

11.3 Authentication & Integrity (AE)

Single scheme ensuring confidentiality + integrity.

Common designs:

- **Encrypt-then-MAC:** Encrypt first, then authenticate ciphertext \rightarrow secure (modification detected before decryption)
- **Encrypt-and-MAC:** Process done independently \rightarrow weak, integrity not linked to ciphertext; some attacks possible if verification skipped
- **MAC-then-Encrypt:** Encrypts both message and tag \rightarrow weak, attacker can modify ciphertext and mislead error handling (e.g., old TLS)

11.4 Why Encrypt-then-MAC is Best

- Verify integrity **before** decryption \rightarrow reject tampered ciphertext early
- Prevents padding oracle attacks, chosen-ciphertext attacks
- Generic composition works with any secure encryption + MAC
- Modern modes (GCM, CCM, ChaCha20-Poly1305) implement this principle

12 Public Key Infrastructure (PKI)

12.1 The Key Distribution Problem

- Public key cryptography enables encryption/signatures without shared secrets
- **Problem:** How to trust that a public key belongs to the claimed identity?
- Example: Alice receives PK_B claiming to be Bob's key. How does she verify?
- Without verification \rightarrow Man-in-the-Middle attacks possible

12.2 Certificates & Certificate Authorities (CA)

Certificate: digitally signed statement binding identity to public key

Certificate structure

- Subject name (e.g., 'alice@example.com' or 'www.bank.com')
- Subject's public key (PK)
- Validity period (not before / not after dates)
- Issuer (CA) name
- CA's digital signature: $\text{Sign}_{SK_{CA}}(H(\text{certificate data}))$

Certificate Authority (CA)

- Trusted third party that verifies identities and issues certificates
- CA's public key (PK_{CA}) is widely known and trusted
- Anyone can verify certificate: $\text{Verify}_{PK_{CA}}(\text{cert}, \text{signature})$

12.3 Certificate Chains & Trust Hierarchy

- **Root CA:** Top-level CA, self-signed certificate
- **Intermediate CA:** Issued by Root CA, issues end-entity certificates
- **End-entity certificate:** Issued to users/servers/devices
- **Chain verification:** End-entity \leftarrow Intermediate \leftarrow Root
- Browsers/OS pre-install trusted root CA certificates

12.4 PKI Security Properties

- **Authenticity:** Certificates bind verified identities to public keys
- **Integrity:** Signatures prevent certificate tampering
- **Trust anchor:** Security relies on protecting root CA private keys
- **Weakest link:** Compromise of any CA in chain breaks trust

13 Password Security

13.1 Storing Passwords

Correct approach: Hash + Salt

- Generate random salt s for each user
- Store $(s, H(\text{password}||s))$
- Verification: recompute $H(\text{input}||s)$ and compare
- **Salt prevents rainbow tables:** each user has different hash even with same password
- Salt can be public (stored with hash)

13.2 Password Attacks

- **Dictionary attack:** Try common passwords (123456, password, qwerty)
- **Brute-force:** Try all possible combinations (slow with bcrypt/Argon2)
- **Credential stuffing:** Reuse leaked passwords from other breaches
- **Phishing:** Social engineering to steal credentials directly

- **Timing attacks:** Measure comparison time to leak password length

13.3 Password Strength

- **Entropy:** $\log_2(\text{possible passwords})$
- Example: 8 random chars (a-z, A-Z, 0-9, symbols) \approx 52 bits
- Trade-off: entropy vs memorability
- **Passphrases:** 'correct horse battery staple' \rightarrow high entropy, memorable
- **Best practice:** password managers generate & store random passwords

14 Man-in-the-Middle (MITM) Attack

14.1 Diffie-Hellman Vulnerability

Key observation: DH provides key agreement but NOT authentication

The Attack

1. Alice sends $A = g^a \mod p$ to Bob
2. **Eve intercepts**, sends $E_1 = g^{r_1} \mod p$ to Bob (pretending to be Alice)
3. Bob sends $B = g^b \mod p$ to Alice
4. **Eve intercepts**, sends $E_2 = g^{r_2} \mod p$ to Alice (pretending to be Bob)
5. Alice computes shared key with Eve: $K_{AE} = E_2^a = g^{ar_2} \mod p$
6. Bob computes shared key with Eve: $K_{BE} = E_1^b = g^{r_1b} \mod p$
7. Eve controls all traffic: Alice $\xleftrightarrow{K_{AE}}$ Eve $\xleftrightarrow{K_{BE}}$ Bob

14.2 Consequences

- Eve decrypts all messages from Alice, re-encrypts for Bob (and vice versa)
- Neither Alice nor Bob detect the attack
- Eve can read, modify, or block any message
- **Complete confidentiality breach despite using DH!**